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Generalized Noether Identities for Nonlocal Transformations

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Starting from the transformation properties of an action integral of a system under local and nonlocal transformations, we derive the generalized Noether identities for a variant system under those transformations. The applications of the theory to the Yang–Mills field with higher order Lagrangian is presented under the Coulomb gauge condition, a new conserved PBRS charge is found which differs from the BRS conserved charge, and another conserved charge connected with nonlocal transformation is also obtained.

1. INTRODUCTION

Local gauge invariance is now a central concept in modern field theory. The classical Noether (1918) identities refer to invariance of the action integral of the system under a local transformation parametrized by r arbitrary functions and their derivatives. If an action integral is invariant under such transformation, then there are r differential identities (Noether identities), which involve the functional derivatives of the action integral. These identities were discussed by Hilbert (1924) and Bergmann (1949) and Anderson and Bergmann (1951) in connection with electrodynamics and general relativity, by Drobot and Rybarski (1958–1959) in connection with hydromechanics, by Sundermyer (1982) in connection with gauge theories, and by others (Li, 1993a) in a general way. A generalization of classical Noether identities for a system with noninvariant action integral under a local transformation was given in Li (1987). A canonical Noether identity in phase space was also developed (Li, 1993b).

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In the quantum theories of the Yang–Mills field (Kuang and Yi, 1980) and the conformal transformation of quantum fields in gauge theories (Fradkin and Palchik, 1984; Palchik, 1985), nonlocal transformations were introduced. The invariant effective Lagrangian under the BRS transformation is not invariant under the gauge transformation alone. Therefore, the investigation of noninvariance properties of a system under local and nonlocal transformations is necessary. Dynamical systems described in terms of higher order Lagrangians obtained by many authors are of much interest in connection with gauge theories, gravity, supersymmetry, string models, and other problems (Li, 1991, 1993c).

In this paper we discuss a more general case for the system with a Lagrangian involving higher order derivatives. Starting from the transformation properties of an action integral of such a system under local and nonlocal transformations we derive the generalized Noether identities (GNI) for a variant system under those transformations. These GNI are integral-differential identities which involve the functional derivatives of the action integral. The applications of the theory to effective higher order Lagrangian for the quantum gauge field in the Coulomb gauge is given, a new conserved PBRS charge is found which differs from the BRS conserved charge, and another conserved charge connected with the nonlocal transformation is also obtained.

2. GENERALIZATION OF NOETHER IDENTITIES

Let us consider a physical field defined by n field functions $\phi^\alpha(x)$ ($\alpha = 1, 2, \ldots, n$) $x = (x^0, x^i)$, $x^0 = t$, $i = 1, 2, 3$. The flat space-time metric is $\eta_{\mu\nu} = \text{diag}(+---)$, $\mu, \nu = 0, 1, 2, 3$. The Lagrangian density of the system, which may involve higher order derivatives of the field functions, is

$$\mathcal{L} = \mathcal{L}(x; \phi^\alpha(x), \ldots, \phi^\alpha_{,\mu(m)}(x), \ldots) \tag{1}$$

where

$$\phi^\alpha_{,\mu(m)}(x) = \partial_{\mu(m)}\phi^\alpha(x) = \underbrace{(\partial_\mu \partial_\nu \cdots \partial_\lambda)}_{m}\phi^\alpha(x) \qquad (m = 1, 2, \ldots, N) \tag{2}$$

The action integral of this system is

$$I = \int_\Omega d^4x \, \mathcal{L}(x, \phi^\alpha, \ldots, \phi^\alpha_{,\mu(m)}, \ldots) \tag{3}$$

Throughout the paper it is supposed that all functions and their derivatives up to required order are smooth enough.

Let us consider the transformation properties of an action of the system under general local and nonlocal transformations, whose infinitesimal transformation is

$$\begin{cases} x^{\mu'} = x^{\mu} + \Delta x^{\mu} = x^{\mu} + R_{\sigma}^{\mu}\epsilon^{\sigma}(x) \\ \phi^{\alpha'}(x') = \phi^{\alpha}(x) + \Delta\phi^{\alpha}(x) \\ \qquad = \phi^{\alpha}(x) + A_{\sigma}^{\alpha}\epsilon^{\sigma}(x) + \int_{\Omega} d^4y\, F(x, y)B_{\sigma}^{\alpha}(y)\epsilon^{\sigma}(y) \end{cases} \tag{4}$$

where

$$R_{\sigma}^{\mu} = r_{\sigma}^{\mu\nu(l)}\partial_{\nu(l)}, \qquad A_{\sigma}^{\alpha} = a^{\mu\nu(m)}\partial_{\nu(m)}, \qquad B_{\sigma}^{\alpha} = b_{\sigma}^{\alpha\nu(n)}\partial_{\nu(n)}$$
$$r_{\sigma}^{\mu\nu(l)} = r_{\sigma}^{\overbrace{\mu\nu\cdots\lambda}^{l}}, \qquad a_{\sigma}^{\mu\nu(m)} = a_{\sigma}^{\overbrace{\mu\nu\cdots\delta}^{m}}, \qquad b_{\sigma}^{\alpha\nu(n)} = b_{\sigma}^{\overbrace{\alpha\nu\cdots\rho}^{n}} \tag{5}$$

and $r_{\sigma}^{\mu\nu(l)}$, $a_{\sigma}^{\mu\nu(m)}$, and $b_{\sigma}^{\alpha\nu(n)}$ are functions of x, ϕ^{α}, $\phi^{\alpha}_{,\mu(m)}$; $\epsilon^{\sigma}(x)$ ($\sigma = 1, 2, \ldots, r$) are arbitrary infinitesimal independent functions; the values of $\epsilon^{\sigma}(x)$ and their derivatives up to required order on the boundary of domain Ω vanish. It is supposed that under the transformation (4) the variation of (3) is given by

$$\delta I = \int_{\Omega} d^4x\, [\partial_{\mu}(\Lambda_{\sigma}^{\mu}\epsilon^{\sigma}(x)) + W] \tag{6}$$

where

$$W = U_{\sigma}\epsilon^{\sigma}(x) + \int_{\Omega} d^4y\, V_{\sigma}(x, y)\epsilon^{\sigma}(y) \tag{7}$$

$$\Lambda_{\sigma}^{\mu} = \lambda_{\sigma}^{\mu\nu(i)}(x)\partial_{\nu(i)}, \qquad U_{\sigma} = U_{\sigma}^{\mu(j)}(x)\partial_{\mu(j)}, \qquad V_{\sigma} = V_{\sigma}^{\mu(k)}(x, y)\partial_{\mu(k)} \tag{8}$$

and $\lambda_{\sigma}^{\mu\nu(i)}$, $U_{\sigma}^{\mu(j)}$, and $V_{\sigma}^{\mu(k)}$ are functions of the x, ϕ^{α}, $\phi^{\alpha}_{,\mu(m)}$. For a weakly quasiinvariant system (Lusanna, 1991), $W \doteq 0$, which means "evaluated on the trajectory of motion." Under the transformation (4), from (3), (4), and (6) we have (Li, 1987)

$$\begin{aligned} \int_{\Omega} d^4x \Bigg\{ & \frac{\delta I}{\delta\phi^{\alpha}} [(A_{\sigma}^{\alpha} - \phi^{\alpha}_{,\mu}R_{\sigma}^{\mu})\epsilon^{\sigma}(x) \\ & + \int_{\Omega} d^4y\, F(x, y)B_{\sigma}^{\alpha}(y)\epsilon^{\sigma}(y)] + \partial_{\mu}(j_{\sigma}^{\mu}\epsilon^{\sigma}(x)) \\ & + \partial_{\mu}\left[\sum_{m=0}^{N-1} \Pi_{\alpha}^{\mu\nu(m)}\partial_{\nu(m)} \int_{\Omega} d^4y\, F(x, y)B_{\sigma}^{\alpha}(y)\epsilon^{\sigma}(y)\right]\Bigg\} \\ = \int_{\Omega} d^4x & \left[U_{\sigma}(x)\epsilon^{\sigma}(x) + \int_{\Omega} d^4y\, V_{\sigma}(x, y)\epsilon^{\sigma}(y)\right] \end{aligned} \tag{9}$$

where

$$\frac{\delta I}{\delta\phi^\alpha} = (-1)^m \partial_{\mu(m)} \mathscr{L}_\alpha^{\mu(m)} \tag{10}$$

$$\mathscr{L}_\alpha^{\mu(m)} = \frac{1}{m!} \sum_{\substack{\text{all permutation}\\ \text{of indices}}} \frac{\partial \mathscr{L}}{\partial \phi^\alpha_{,\mu(m)}} \tag{11}$$

$$\Pi_\alpha^{\mu\nu(m)} = \sum_{l=0}^{N-(m+1)} (-1)^l \partial_{\lambda(l)} \mathscr{L}_\alpha^{\mu\nu(m)\lambda(l)} \tag{12}$$

$$j_\sigma^\mu = \mathscr{L} R_\sigma^\mu + \sum_{m=0}^{N-1} \Pi_\alpha^{\mu\nu(m)} \partial_{\nu(m)} (A_\sigma^\alpha - \phi^\alpha_{,\lambda} R_\sigma^\lambda) - \Lambda_\sigma^\mu \tag{13}$$

Using the Gauss theorem, we can get the term of the integral of $\partial_\mu(j_\sigma^\mu \epsilon^\sigma)$ in (9) to vanish because of the boundary conditions of $\epsilon^\sigma(x)$, after which we functionally differentiate (9) with respect to $\epsilon^\rho(z)$ ($\rho = 1, 2, \ldots, r$), we obtain

$$\begin{aligned} &\tilde{A}_\rho^\alpha(z)\left(\frac{\delta I}{\delta\phi^\alpha(z)}\right) - \tilde{R}_\rho^\mu(z)\left(\phi^\alpha_{,\mu}(z)\,\frac{\delta I}{\delta\phi^\alpha(z)}\right) \\ &\quad + \int_\Omega d^4x\, \tilde{B}_\rho^\alpha(z)\left(F(x, z)\,\frac{\delta I}{\delta\phi^\alpha(x)}\right) \\ &\quad + \int_\Omega d^4x\, \tilde{B}_\rho^\alpha(z)\left\{\partial_\mu\left[\sum_{m=0}^{N-1} \Pi_\alpha^{\mu\nu(m)} \partial_{\nu(m)} F(x, z)\right]\right\} \\ &= \tilde{U}_\rho(z) + \int_\Omega d^4x\, \tilde{V}_\rho(x, z) \end{aligned} \tag{14}$$

where $\tilde{A}_\rho^\alpha$, $\tilde{B}_\rho^\alpha$, $\tilde{R}_\rho^\mu$, $\tilde{U}_\rho$, and $\tilde{V}_\rho$ are the adjoint operators with respect to A_ρ^α, B_ρ^α, R_ρ^μ, U_ρ, and V_ρ, respectively (Li, 1987). Therefore, we have the following generalized second Noether theorem:

If the variation of an action integral (3) is given by (6) under the transformation (4), then there are r identities (14) between the functional derivatives $\delta I/\delta\phi^\alpha$ and their derivatives up to some fixed order.

These identities (14) are called generalized Noether identities (GNI) and are integral-differential ones. In case of invariance ($W = 0$) the right-hand side of (14) equals zero. Thus, we have identity relations between the functional derivatives and their derivatives:

$$\begin{aligned} &\tilde{A}_\rho^\alpha(z)\left(\frac{\delta I}{\delta\phi^\alpha(z)}\right) - \tilde{R}_\rho^\mu(z)\left(\phi^\alpha_{,\mu}\,\frac{\delta I}{\delta\phi^\alpha(z)}\right) \\ &\quad + \int_\Omega d^4x\, \tilde{B}_\rho^\alpha(z)\left(F(x, z)\,\frac{\delta I}{\delta\phi^\alpha(z)}\right) \end{aligned}$$

$$+ \int_{\Omega} d^4x\, \tilde{B}^{\alpha}_{\rho}(z) \left\{ \partial_{\mu} \sum_{m=0}^{N-1} \Pi^{\mu\nu(m)}_{\alpha} \partial_{\nu(m)} F(x, z) \right] \Bigg\}$$

$$= 0 \qquad (\rho = 1, 2, \ldots, r) \tag{15}$$

These identities are valid whether the equations of motion are satisfied or not.

3. WEAK CONSERVATION LAWS

Let us now consider a system with noninvariant action integral under local transformation. As is well known, in massive Yang–Mills theory, the Lagrangian is not invariant under gauge transformation; interaction of massive Fermi fields with gauge fields is not invariant under the chirality transformation of the Fermi fields; the effective Lagrangian with Faddeev–Popov ghost fields is not invariant under the gauge transformation alone; the invariance is restored only under the BRS transformation. Therefore, for the Lagrangian of a system which is not invariant under the local transformation, the discussion of the transformation properties is necessary. Let us put $\Delta x^{\mu} = 0$ and $F(x, y) = 0$ in (4), as is usually done in the discussion of gauge transformation,

$$\phi^{\alpha'}(x) = \phi^{\alpha}(x) + A^{\alpha}_{\sigma}\epsilon^{\sigma}(x) = \phi^{\alpha}(x) + (a^{\alpha}_{\sigma} + a^{\alpha\mu}_{\sigma}\partial_{\mu})\epsilon^{\sigma}(x) \tag{16}$$

where a^{α}_{σ} and $a^{\alpha\mu}_{\sigma}$ are functions of x and ϕ^{α}. Under the transformation (16), suppose that the variation of the Lagrangian is given by

$$\delta\mathcal{L} = (U_{\sigma} + U^{\mu}_{\sigma}\partial_{\mu} + U^{\mu\nu}_{\sigma}\partial_{\mu}\partial_{\nu} + U^{\mu\nu\lambda}_{\sigma}\partial_{\mu}\partial_{\nu}\partial_{\lambda})\epsilon^{\sigma}(x) \tag{17}$$

where U_{σ}, U^{μ}_{σ}, $U^{\mu\nu}_{\sigma}$, and $U^{\mu\nu\lambda}_{\sigma}$ are functions of x, ϕ, and $\phi^{\alpha}_{,\mu}$. For example, a system with second-order Lagrangian for Yang–Mills theories belongs to this category. Under the transformation (16), from the variation of the action integral (3), one has the basic identity

$$\frac{\delta I}{\delta\phi^{\alpha}} a^{\alpha}_{\sigma}\epsilon^{\sigma}(x) + \frac{\delta I}{\delta\phi^{\alpha}} a^{\alpha\mu}_{\sigma}\partial_{\mu}\epsilon^{\sigma}(x)$$

$$+ \partial_{\mu}\left[\sum_{m=0}^{N-1} \Pi^{\mu\nu(m)}_{\alpha}\partial_{\nu(m)}A^{\alpha}_{\sigma}\epsilon^{\sigma}(x)\right]$$

$$= (U_{\sigma} + U^{\mu}_{\sigma}\partial_{\mu} + U^{\mu\nu}_{\sigma}\partial_{\mu}\partial_{\nu} + U^{\mu\nu\lambda}_{\sigma}\partial_{\mu}\partial_{\nu}\partial_{\lambda})\epsilon^{\sigma}(x) \tag{18}$$

The GNI (14) is this case becomes

$$a^{\alpha}_{\sigma}\frac{\delta I}{\delta\phi^{\alpha}} - \partial_{\mu}\left(a^{\alpha\mu}_{\sigma}\frac{\delta I}{\delta\phi^{\alpha}}\right) = U_{\sigma} - \partial_{\mu}U^{\mu}_{\sigma} + \partial_{\mu}\partial_{\nu}U^{\mu\nu}_{\sigma} - \partial_{\mu}\partial_{\nu}\partial_{\lambda}U^{\mu\nu\lambda} \tag{19}$$

Multiplying the identities (19) by $\epsilon^{\sigma}(x)$, summing with index σ from 1 to r, and subtracting the result from the basic identity (18), if the indices μ, ν of the coefficients $U_\sigma^{\mu\nu}$ are symmetrical and the indices μ, ν and μ, λ of the coefficients $U_\sigma^{\mu\nu\lambda}$ are also symmetrical, then we obtain

$$\partial_\mu J^\mu = 0 \tag{20a}$$

$$\begin{aligned} J^\mu = & \sum_{m=0}^{N-1} \Pi_\alpha^{\mu\nu(m)} \partial_{\nu(m)} A_\sigma^\alpha \epsilon^\sigma(x) + a_\sigma^{\alpha\mu} \frac{\delta I}{\delta\phi^\alpha} \epsilon^\sigma(x) \\ & - U_\sigma^\mu \epsilon^\sigma(x) + (\partial_\nu U_\sigma^{\mu\nu}) \epsilon^\sigma(x) \\ & - U_\sigma^{\mu\nu} \partial_\nu \epsilon^\sigma(x) + U_\sigma^{\mu\nu\lambda} \partial_\nu \partial_\lambda \epsilon^\sigma(x) + (\partial_\nu \partial_\lambda U_\sigma^{\mu\nu\lambda}) \epsilon^\sigma(x) \\ & - (\partial_\nu U_\sigma^{\mu\nu\lambda}) \partial_\lambda \epsilon^\sigma(x) \end{aligned} \tag{20b}$$

The equations of motion were not used in deducing (20a), (20b), hence these expressions (strong conservation law) holds whether the equations of motion are satisfied or not.

If $\epsilon^\sigma(x) = \epsilon_0^\rho \zeta_\rho^\sigma(x)$ in the transformation (16), where ϵ_0^ρ are numerical parameters and $\zeta_\rho^\sigma(x)$ are functions of x, ϕ^α, and $\phi^\alpha_{,\mu(m)}$, then, along the trajectory of the motion, $\delta I/\delta\phi^\alpha = 0$, we can obtain the weak conservation laws

$$\partial_\mu J_\rho^\mu = 0 \tag{21a}$$

$$\begin{aligned} J_\rho^\mu = & \sum_{m=0}^{N-1} \Pi_\alpha^{\mu\nu(m)} \partial_{\nu(m)} A_\sigma^\alpha \zeta_\rho^\sigma - U_\sigma^\mu \zeta_\rho^\sigma + (\partial_\nu U_\sigma^{\mu\nu}) \zeta_\rho^\sigma - U_\sigma^{\mu\nu} \partial_\nu \zeta_\rho^\sigma \\ & + U_\sigma^{\mu\nu\lambda} \partial_\nu \partial_\lambda \zeta_\rho^\sigma + (\partial_\nu \partial_\lambda U_\sigma^{\mu\nu\lambda}) \zeta_\rho^\sigma - (\partial_\nu U_\sigma^{\mu\nu\lambda}) \partial_\lambda \zeta_\rho^\sigma \end{aligned} \tag{21b}$$

Thus, one can see that for certain cases the GNI may be converted into (weak) conservation laws along the trajectory of motion even if the Lagrangian $\mathscr{L}$ of the system is not invariant under the specific transformation (16). This algorithm for deriving the conservation laws differs from the classical first Noether theorem, where invariance under a global transformation implies the conservation laws. This gives us a new method to find the conservation laws for a system.

4. PBRS CHARGE

Yang–Mills theory with a higher order Lagrangian is given by (Gitman and Tyutin, 1990)

$$\mathscr{L}_{YM} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} - \kappa D_{b\lambda}^a F^{b\mu\nu} D_e^{a\lambda} F_{\mu\nu}^e \tag{22}$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^a_{bc} A^b_\mu A^c_\nu \tag{23}$$

$$D^a_{b\mu} = \delta^a_b \partial_\mu + f^a_{bc} A^c_\mu \tag{24}$$

and f^a_{bc} are structure constants of the gauge group. The Lagrangian (22) is invariant under following gauge transformation:

$$\delta A^a_\mu = D^a_{b\mu} \epsilon^b(x) \tag{25}$$

where $\epsilon^b(x)$ are arbitrary functions. This local invariance of Lagrangian $\mathscr{L}_{\mathrm{YM}}$ implies that the system is subject to some inherent phase space constraint (Li, 1993a), and one can find that all constraints are first class. In the quantum theory for a system with first-class constraints, the gauge condition must be chosen. Through a transformation of the generating functional of the Green function for the Lagrangian (22) in the Coulomb gauge one can obtain the effective Lagrangian (Gitman and Tyutin, 1990)

$$\mathscr{L}_{\mathrm{eff}} = \mathscr{L}_{\mathrm{YM}} + \mathscr{L}_{\mathrm{gh}} - \frac{1}{2\alpha_0} (\partial^i A^a_i)^2 \tag{26}$$

where

$$\mathscr{L}_{\mathrm{gh}} = -\partial^i \overline{C}^a(x) D^a_{bi} C^b(x) \tag{27}$$

$\overline{C}^a(x)$, $C^b(x)$ are ghost fields, and α_0 is a parameter. The Lagrangian (25) is invariant under the following BRS transformation:

$$\begin{cases} \partial A^a_\mu = D^a_{b\mu} C^b \tau \\ \delta C^a = \dfrac{1}{2} f^a_{be} C^b C^e \tau \\ \delta \overline{C}^a = -\dfrac{1}{\alpha_0} \partial^\mu A^a_\mu \tau \end{cases} \tag{28}$$

where τ is Grassman's parameter. According the classical first Noether theorem, a consequence of the BRS invariance of the effective Lagrangian is the presence of conserved current J^ν in Coulomb gauge,

$$\begin{aligned} J^\nu &= \frac{\partial \mathscr{L}_{\mathrm{eff}}}{\partial A^a_{\mu,\nu}} D^a_{b\mu} C^b + \frac{\partial \mathscr{L}_{\mathrm{eff}}}{\partial A^a_{\mu,\nu\rho}} \partial_\rho (D^a_{b\mu} C^b) - \partial_\rho \left(\frac{\partial \mathscr{L}_{\mathrm{eff}}}{\partial A^a_{\mu,\nu\rho}} \right) D^a_{b\mu} C^b \\ &\quad + \frac{\partial \mathscr{L}_{\mathrm{eff}}}{\partial C^a_{,\nu}} \delta C^a + \frac{\partial \mathscr{L}_{\mathrm{eff}}}{\partial \overline{C}^a_{,\nu}} \delta \overline{C}^a \\ &= J^\nu_1 - \frac{1}{2} \partial^\nu \overline{C}^a f^a_{be} C^b C^e + D^{a\nu}_b C^b \partial^0 A^a_0 \end{aligned} \tag{29}$$

where

$$J_1^\nu = \frac{1}{\kappa^2}\left\{\left[f_{ea}^m A^{e\rho}\partial_\rho F^{m\mu\nu} + f_{ae}^m f_{fd}^m A_\lambda^e A^{f\lambda} F^{d\mu\nu}\right.\right.$$
$$\left. - \partial_\rho(\partial^\rho F^{a\mu\nu} + f_{ed}^a A^{e\rho}F^{d\mu\nu})\right] D_{b\mu}^a C^b$$
$$\left. + (\partial^\rho F^{a\mu\nu} + F_{ed}^a A^{e\rho}F^{d\mu\nu})\partial_\rho(D_{b\mu}^a C^b)\right\} - F^{a\nu\mu}D_{b\mu}^a C^b \tag{30}$$

which implies the conserved BRS charge

$$Q = \int d^3x\, J^0 = \int d^3x \left[J_1^0 - \frac{1}{2} f_{be}^a \partial^0 \overline{C}^a C^b C^e + D_b^{a0} C^b \partial^0 A_0^a\right] \tag{31}$$

Now let us consider only the transformation of the Yang–Mills fields, fixing the ghost fields in the BRS transformation, i.e.

$$\begin{cases} \delta A_\mu^a = D_{b\mu}^a C^b \tau \\ \delta C^a = \delta \overline{C}^a = 0 \end{cases} \tag{32}$$

Under the transformation (32), the effective Lagrangian (26) is variant, and

$$\delta \mathcal{L}_{\text{eff}} = F(\theta) + U_a^\mu \partial_\mu \theta^a + U_a \partial^\mu \partial_\mu \theta^a$$
$$= F(\theta) - \frac{1}{\alpha_0}(\partial^i A_i^a)\partial^\mu\partial_\mu\theta^a - \frac{1}{\alpha_0} f_{bc}^a (\partial^i A_i^b) A_j^c \partial^j \theta^a$$
$$- f_{bc}^a \partial^\mu \overline{C}^c C^b \partial_\mu \theta^a \tag{33}$$

where $\theta^a = C^a\tau$, and $F(\theta)$ does not contain the terms of the derivatives of θ^a. Along the trajectory of motion, from (21) and (33), we obtain the following conserved current J_P^ν in Coulomb gauge:

$$J_P^\nu = J_1^\nu + f_{bc}^a \partial^\nu \overline{C}^c C^b C^a \tag{34}$$

Thus, we obtain the conserved PBRS charge (P stands for "partial")

$$Q_P = \int d^3x\, J_P^0 = \int d^3x\, [J_1^0 + f_{bc}^a \partial^0 \overline{C}^c C^b C^a] \tag{35}$$

This conserved PBRS charge differs significantly from the conserved BRS charge (31).

As is well known, BRS charge annihilates the vacuum; the conserved PBRS charge may also impose some supplementary conditions on physical states as well as BRS charge and ghost charge.

5. NONLOCAL TRANSFORMATION AND CONSERVATION LAWS

It is easy to check that the terms $\mathcal{L}_{YM}$ and $\mathcal{L}_{gh}$ in the effective Lagrangian (26) for higher order Yang–Mills theory is invariant under the following transformation:

$$A^{a'}_{\mu}(x) = A^{a}_{\mu}(x) + D^{a}_{\sigma\mu}\epsilon^{\sigma}(x) \tag{36a}$$

$$C^{a'}(x) = C^{a}(x) + ig(T_{\sigma})^{a}_{b}C^{b}(x)\epsilon^{\sigma}(x) \tag{36b}$$

$$\partial^{\mu}\overline{C}^{a}(x') = \partial^{\mu}\overline{C}^{a}(x) - ig\partial^{\mu}\overline{C}^{b}(x)(T_{\sigma})^{a}_{b}\epsilon^{\sigma}(x) \tag{36c}$$

where T_{σ} ($\sigma = 1, 2, \ldots, M$) are representation matrices of the generators of the gauge group, and $\epsilon^{\sigma}(x)$ are infinitesimal arbitrary functions. (36c) can be written as

$$\begin{aligned}\overline{C}^{a'}(x) &= \overline{C}^{a}(x) - ig\overline{C}^{b}(x)(T_{\sigma})^{a}_{b}\epsilon^{\sigma}(x) \\ &\quad + \frac{ig}{\Box}\partial_{\mu}[\overline{C}^{b}(x)(T_{\sigma})^{a}_{b}\partial^{\mu}\epsilon^{\sigma}(x)]\end{aligned} \tag{36c$'$}$$

where $\Box = \eta^{\mu\nu}\partial_{\mu}\partial_{\nu}$. (36c′) can be reduced to (Kuang and Yi, 1980)

$$\begin{aligned}\overline{C}^{a'}(x) &= \overline{C}^{a}(x) - ig\overline{C}^{b}(x)(T_{\sigma})^{a}_{b}\epsilon^{\sigma}(x) \\ &\quad + ig\int d^{4}y\, \Delta_{0}(x, y)\partial_{\mu}[\overline{C}^{b}(y)(T_{\sigma})^{a}_{b}\partial^{\mu}\epsilon^{\sigma}(y)]\end{aligned} \tag{36c$''$}$$

where

$$\Box\Delta_{0}(x, y) = i\delta^{(4)}(x - y) \tag{37}$$

The transformation (36c″) is a nonlocal one. Under the transformation (36a), (36b) and (36c″), from (14) and (26), we obtain

$$\begin{aligned}&\tilde{D}^{a}_{\rho\mu}\left(\frac{\delta I_{\text{eff}}}{\delta A^{a}_{\mu}(z)}\right) + ig(T_{\rho})^{a}_{b}\frac{\delta I_{\text{eff}}}{\delta C^{a}(z)}C^{b}(z) - ig\overline{C}^{b}(z)(T_{\rho})^{a}_{b}\frac{\delta I_{\text{eff}}}{\delta \overline{C}^{a}(z)} \\ &\quad + \int_{\Omega} d^{4}x\, \tilde{B}^{a}_{\rho}(z)\left[\partial_{\mu}\left(\frac{\partial \mathcal{L}_{\text{eff}}}{\partial \overline{C}^{a}_{,\mu}}\right)\Delta_{0}(x, z)\right] = \frac{1}{\alpha_{0}}\tilde{D}^{a}_{\rho i}[\partial^{i}(\partial^{k}A^{a}_{k})]\end{aligned} \tag{38}$$

where

$$\tilde{D}^{a}_{\rho\mu} = -\delta^{a}_{\rho}\partial_{\mu} + f^{a}_{\rho c}A^{c}_{\mu} \tag{39}$$

$$B^{a}_{\rho}(z) = ig\partial_{\mu}[\overline{C}^{b}(z)(T_{\rho})^{a}_{b}\partial^{\mu}] \tag{40}$$

Under the Coulomb gauge, along the trajectory of motion, from (38) we obtain

$$\partial^{z_\mu} \int_\Omega d^4x\, \overline{C}^b(z)(T_\rho)^a_b \partial_{z_\mu} \left[\partial_{x_\nu} \left(\frac{\partial \mathcal{L}_{\text{eff}}}{\partial \overline{C}^a_{,x_\nu}} \right) \Delta_0(x, z) \right] = 0 \tag{41}$$

which implies the conserved charge

$$Q' = \int_V d^3z \int_\Omega d^4x\, \overline{C}^b(z)(T_\rho)^a_b \partial_{z0} \left[\partial_{x_\nu} \left(\frac{\partial \mathcal{L}_{\text{eff}}}{\partial \overline{C}^a_{,x_\nu}} \right) \Delta_0(x, z) \right] = \text{const} \tag{42}$$

Substituting (26) into (42), we get

$$Q' = \int_V \int_\Omega d^3z\, d^4x\, \overline{C}^b(z)(T_\rho)^a_b (\partial_\nu D^{a\nu}_e C^e) \partial_{z_0} \Delta_0(x, z) = \text{const} \tag{43}$$

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